# LEFT-HANDED LUMPED TRANSMISSION LINES AND LEFT-HANDED SMALL IMPEDANCE TRANSFORMERS

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**ABSTRACT:** *Based on the concept that a left-handed transmission line section has a negative electrical length, two* Π*-type lumped equivalent circuits are presented, and it is newly suggested that a low-pass filter type can be its lumped-equivalent circuit under a certain condition. Applying the negative electrical length concept to Smith chart, left-handed small impedance transformers are derived and two are validated by measurements, adding 180° right-handed transmission lines to the impedance transformers. The measured results are in good agreements with predictions, showing that insertion and return losses are −0.23 and −22.5 dB, respectively.* © 2008 Wiley Periodicals, Inc. Microwave Opt Technol Lett 50: 2269–2271, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.23676

**Key words:** *left-handed transmission lines; left-handed lumped equivalent circuits; left-handed transmission line behavior on Smith chart; left-handed small impedance transformers; left-handed CVTs (LHCVTs)*

## 1. INTRODUCTION

Meta material (metamaterial) is particularly used when the resulting material has properties not found in naturally-formed substances. Nearly all materials encountered in optics or microwaves have positive values for both permittivity $\varepsilon$ and permeability $\mu$. However, many metals such as silver and gold have negative $\varepsilon$ at visible wavelengths and a material having either negative $\varepsilon$ or $\mu$ (not both) is opaque to electromagnetic radiation. The metamaterials have both $\varepsilon < 0$ and $\mu < 0$ but the product $\varepsilon\,\mu$ is positive and such substances can transmit energy.

A Russian physicist Veselago [1] proved the phenomena of the metamaterials and the metamaterials are more commonly referred to as left-handed materials. The left-handed materials having received substantial attention in the scientific and engineering communities and are promising for a diversity of optical/microwave applications, such as bandpass filters, microwave couplers, new types of beam steerers, modulators, superlenses, and so on.

In this letter, Π-type lumped equivalent circuits of left-handed transmission line sections are derived, depending on the electrical lengths. Based on the derived results, it is newly suggested that low-pass filter types can also be their equivalent circuits, while conventional lumped equivalent circuits are only high-pass filter types [2]. Applying the negative electrical length concept to Smith chart, left-handed small impedance transformers, LHCCTs, (left-handed constant conductance impedance transformers) are derived and two LHCCTs are validated by measurements. The measured results are in good agreements with predictions, showing insertion and return losses at a center frequency of 3 GHz that are −0.23 and −22.5 dB, respectively.

## 2. LEFT-HANDED (LH) TRANSMISSION LINES

Although the microwave properties of a transparent material are fully specified by the parameters $\varepsilon$ and $\mu$, in practice the refractive index $N$ is often used. The refractive index $N$ is defined as

$$N = \pm\sqrt{\varepsilon\mu} = \pm\frac{c\beta}{\omega}, \tag{1}$$

where $\beta$, $\omega$, and $c$ are phase constant, angular frequency, and light velocity, respectively. For a negative refractive index, $\beta$ should be negative due to both positive $\omega$ and $c$, which leads to a negative electrical length. Figure 1 shows transmission line sections and their lumped equivalent circuits where right handed (RH) transmission lines are in Figures 1(a) and 1(b) and LH transmission lines in Figures 1(c) and 1(d). In this case, the electrical lengths $\Theta_R$ and $\Theta_L$ are defined as $\beta_R l$ and $\beta_L l$ where $l$ is a transmission line length, the subscripts $R$ and $L$ denote the right- and left-handed materials and $\beta_L$ is once negative. When $\Theta_R$ is less than and equal to 180°, the elements in Figure 1(a) [3] are

$$\omega L_p = Z_0 \sin\Theta_R,\ \omega C_P = \frac{1}{Z_0}\tan\frac{\Theta_R}{2}. \tag{2}$$

When $\Theta_R$ is greater than 180° and less than 360°, those in Figure 1(b) are

$$\omega L_P{}' = -Z_0 \sin\Theta_R,\ \omega C_P{}' = -\frac{1}{Z_0}\tan\frac{\Theta_R}{2}. \tag{3}$$

In a similar way, those of the LH transmission line in Figures 1(c) and 1(d) can be derived using

$$\omega^2 L_p C_n = \omega^2 L_n C_p = 1,$$

$$\omega^2 L_p{}' C_n{}' = \omega^2 L_n{}' C_p{}' = 1 \tag{4}$$

In (2)–(4), the subscripts $p$ and $n$ indicate the positive and negative indexes, or, right and left-handed transmission lines. All the conventional lumped-element equivalent circuits have series capaci-

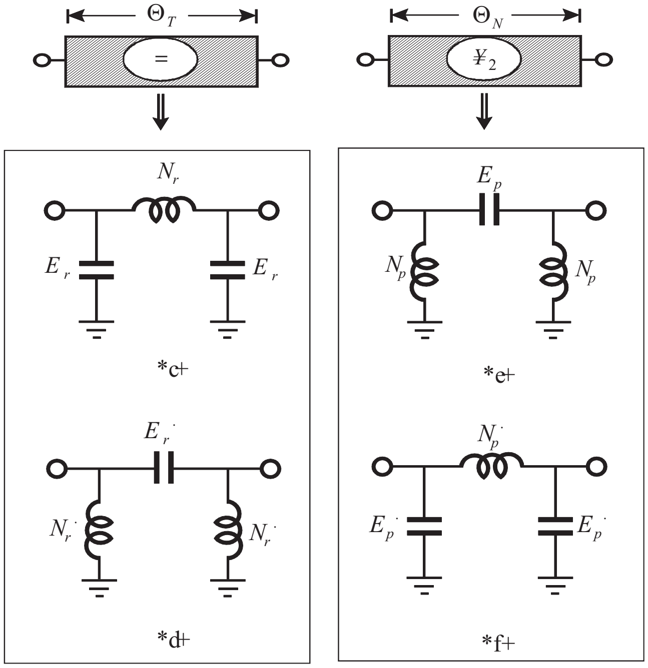

**Figure 1** Transmission line sections and their equivalent circuits. (a) RH transmission line with $0 \leq \Theta_R \leq 180°$, (b) RH transmission line with $180° < \Theta_R < 360°$, (c) LH transmission line with $0 \leq |\Theta_L| \leq 180°$, (d) LH transmission line with $180° < |\Theta_L| < 360°$

tances between two ports [2]. However, Figure 1 shows that the low-pass filter type having a series inductance in Figure 1(d) can be a left-handed equivalent circuit.

## 3. LHCCTS

Using the negative electrical length concept, LH small impedance transformers can be obtained. Figure 2 shows an admittance Smith chart to obtain impedance transformers to transform 100 Ω into 50 Ω. The admittance Smith chart is normalized to 50 Ω and therefore 100 Ω and 50 Ω correspond to 0.5 and unity on the Smith chart. To transform the value of 0.5 into unity, a λ/4 impedance transformer is possible and expressed as a blue colored solid half circle on the upper half plane.

To reduce its size, the value of 0.5 should be moved into a complex value along a constant conductance circle or along a constant VSWR circle [4]. Moving it along the constant conductance circle results in CCTs (constant conductance impedance transformers). One of CCTs is well illustrated in Figure 2 as a pink colored solid locus passing through *m*. Those so far explained are RHCCTs (right-handed constant conductance impedance transformers). In the case of left-handed (RH) impedance transformers, only difference is the moving direction because of the negative electrical length. In the lower half plane of the Smith chart in Figure 2, a blue colored dotted half circle is a LH λ/4 impedance transformer and a pink colored dotted locus passing through *m*′ a LHCCT.

Figure 3 shows RH and LHCCTs to transform a real impedance $Z_L$ into $Z_r$ where $Z_a$, $Z_b$, and $Z_{La}$ are positive real characteristic impedances, $\Theta_{Ra}$, $\Theta_{Rb}$, and $\Theta_{Ls}$ are positive and usually less than 90°, while $\Theta_{La}$ and $\Theta_{Lb}$ in Figure 3(b) are negative and $|\Theta_{La}|$ and $|\Theta_{Lb}|$ usually less than 90°. There are so many cases to transform a real impedance $Z_L = 100\ \Omega$ into another real impedance $Z_r = 50\ \Omega$ and Table 1 gives several. In the case of LHCCTs in Figure 3(b), an open stub with a negative length $\Theta_{La}$ can be realized with a distributed transmission line. An impedance looking into the open stub of the LHCCT, $Z_{L\text{stub}}$ in Figure 3(b) is

$$Z_{Lstub} = -jZ_a \cot\Theta_{La} = jZ_a \cot\Theta_{Ra} = jZ_{La} \tan\Theta_{Ls}, \qquad (5)$$

where $\Theta_{La} = -\Theta_{Ra}$ is used. Equation (5) indicates that the open-stub with a negative electrical length becomes inductance

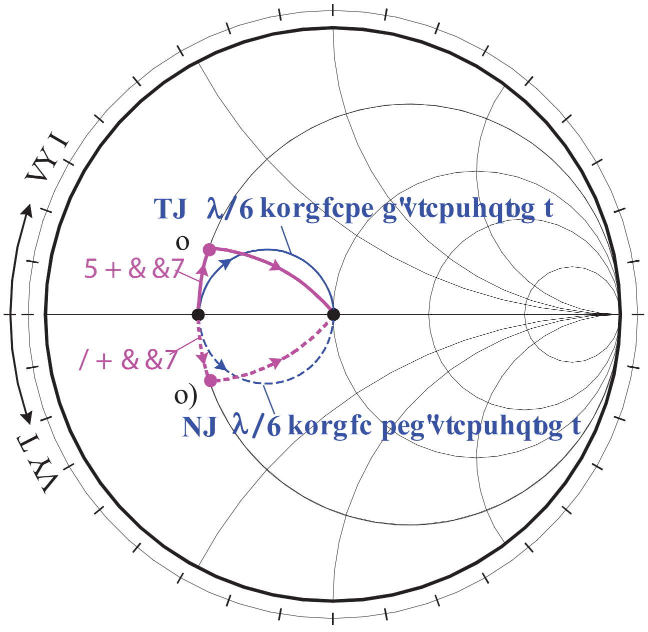

**Figure 2** An admittance Smith chart showing left-handed impedance transformers. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

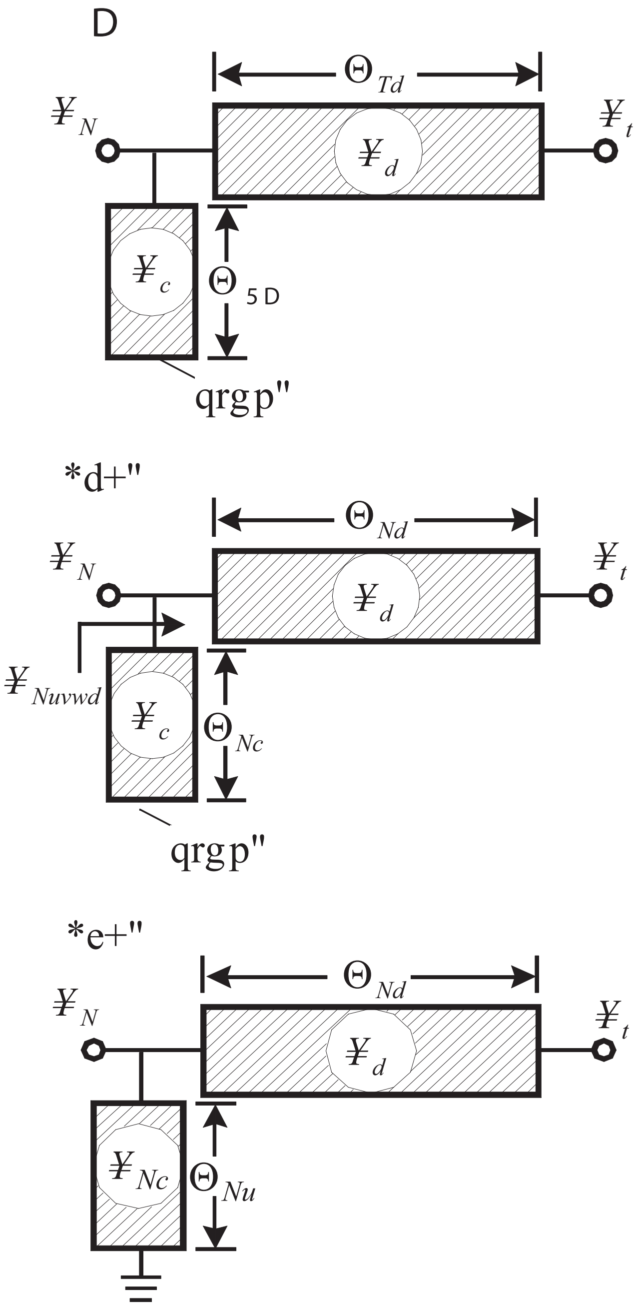

**Figure 3** RH and LHCCTs (a) RHCCT (b) LHCCT (c) LHCCT for realization

**TABLE 1 Several Cases of RH and LHCCTs With $Z_L$ = 100Ω and $Z_r$ = 50Ω**

| | | | | | |
|---|---|---|---|---|---|
| $\Theta_{Ra}$ | 22.76 | 16.1 | 10.3 | 5.02 | 0 |
| $\Theta_{La}$ | −22.76 | −16.1 | −10.3 | −5.02 | 0 |
| $\Theta_{Rb}$ | 24.63 | 45 | 61.08 | 75.8 | 90 |
| $\Theta_{Lb}$ | −24.63 | −45 | −61.08 | −75.8 | −90 |
| $Z_b$ | 129.9 | 86.6 | 75.9 | 71.8 | 70.71 |

$Z_a = 50\Omega$, $\Theta_{La} = -\Theta_{Ra}$ and $\Theta_{Lb} = -\Theta_{Rb}$

and can be realized with a short stub with both positive $Z_{La}$ and $\Theta_{Ls}$ in Figure 3(c). The Eq. (5) yields

$$\Theta_{Ls} = \arctan\left(\frac{Z_a}{Z_{La}}\cot\Theta_{Ra}\right) \tag{6}$$

A transmission line section with a negative electrical length $\Theta_{Lb}$ in Figure 3(b) can be realized with lumped elements or distributed elements combined with lumped elements [2] but can not be realized only with distributed elements. So, if a 180° transmission line section is added to after that with $\Theta_{Lb}$ in Figure 3(b) and 180° is subtracted from the measured phase responses, then the correct measurement results will be obtained. Two LHCCTs with $\Theta_{Ra} = 16.1°$ and $\Theta_{Ra} = 10.3°$ given in Table 1 are designed at a center frequency of 3 GHz and fabricated on a substrate ($\varepsilon_r = 3.5$ and $H = 30\ mil$). In both cases, the $Z_{La}$ in Figure 3(c) is chosen as 80Ω and its resulting lengths of $\Theta_{Ls}$ are 65.21° and 73.78° for $\Theta_{Ra} = 16.1°$ and $\Theta_{Ra} = 10.3°$, respectively. Figure 4 shows photographs of the two fabricated LHCCTs and those with $\Theta_{Ra} = 16.1°$ and $\Theta_{Ra} = 10.3°$ are in Figures 4(a) and 4(b), respectively. Since the impedance transformers are terminated in 100Ω and 50Ω, additional impedance transformers are needed for the measurements. "Ipt" written on the photographs are λ/4 impedance transformers to transform 100Ω into another 50Ω. Figure 5 compares the measured scattering parameters with predictions where good agreements are shown between them. The measured insertion and return losses at 3 GHz are −0.23 and −22.5 dB, respectively.

## 4. CONCLUSION

Using the concept that a negative refractive index $N$ gives a negative electrical length in the transmission line, Π-type lumped equivalent circuits are derived and it is newly suggested that a low-pass type lumped-element circuit can also be another equivalent circuit of a left-handed transmission line section. Applying the negative electrical length concept to a Smith chart, LHCCTs (left-handed constant conductance impedance transformers) are derived and two are validated by measurements. The ideas, that the low-pass filter type can be another lumped element equivalent circuit of left-handed transmission line sections and the negative electrical length can be applied to a Smith chart, are very important and will be used to design various components such as ring hybrids, branch-line couplers, directional couplers, and so on.

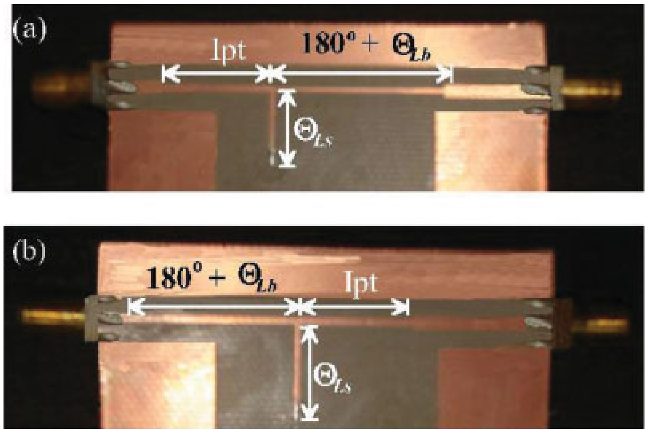

**Figure 4** Photographs of the LHCCTs. (a) LHCCT with $\Theta_{Ra} = 16.1°$ (b) LHCCT with $\Theta_{Ra} = 10.3°$. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

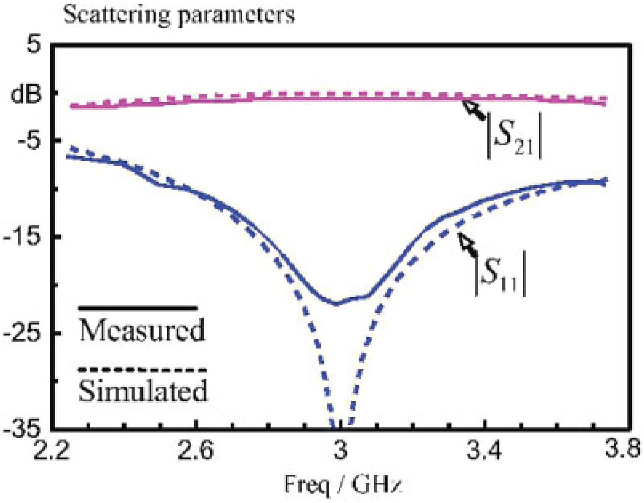

**Figure 5** Measured results are compared with simulated ones. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

**REFERENCES**

1. V. Veselago, The electrodynamics of substances with simultaneously negative values of $\varepsilon$ and $\mu$, Sov Phys Lisp 10 (1968), 509–514.
2. H. Okabe, C. Caloz, and T. Itoh, A compact enhanced-bandwidth hybrid ring using an artificial lumped-element left-handed transmission line section, IEEE Trans Microwave Theory Tech 52 (2004), 798–804.
3. H.-R. Ahn, Asymmetric passive components in microwave integrated circuits, Wiley, New York, 2006, p. 78.
4. H.-R. Ahn and I. Wolff, General design equations of three-port power dividers, small-sized impedance transformers, and their applications to small-sized three-port 3-dB power dividers, IEEE Trans Microwave Theory Tech 49 (2001), 1277–1288.

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# A MINIATURIZED PARALLEL-COUPLED MICROSTRIP FILTER USING OVER-COUPLED END STAGES WITH SYMMETRICAL TAPPED-LINE STRUCTURE

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**ABSTRACT:** *In this article, a miniaturized parallel-coupled microstrip filter is presented. The miniaturized design saves 80% circuit dimension compared to the conventional parallel-coupled over-coupled end stages filter with the symmetrical tapped-line structure. Several design methods and miniaturization techniques were involved in this study to achieve* $5f_0$ *stopband suppression, including symmetrical tapped-line, stepped-impedance structure, bended and folded of transmission lines, and the coupling effect of microstrip-lines. Also the asymmetrical tapped-line structure is fabricated to achieve* $6f_0$ *stopband suppression. Good agreements between the simulated and measured results were also observed in all*