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CSE-433 Logic in Computer Science 2011
Midterm exam

- This is a closed-book exam. No other material is permitted.
- It consists of 4 problems worth a total of 100 points.
- There are 11 pages in this exam.
- You have one and a half hours for this exam.

	Problem 1	Problem 2	Problem 3	Problem 4	Total
Score					
Max	EC	35	25	40	100

제 1 절 대학생활 [Extracredit]

1.1 전산논리

이번학기 전산논리 과목을 한 단어로 요약하면 무엇인가요?

제 2 절 Propositional logic [35pts]

Question 1. [5 pts] Write the introduction and elimination rules for disjunction $\vee$ using truth judgments of the form $C\ true$.

Question 2. [5 pts] Use the judgments in Question 1 and show that local soundness holds for disjunction $\vee$.

Question 3. [5 pts] Use the inference rules in Question 1 and show that local completeness holds for disjunction $\vee$.

Question 4. [5 pts] Write the introduction and elimination rules for disjunction $\vee$ using hypothetical judgments of the form $\Gamma \vdash C\ true$ where Γ is a collection of antecedents.

Question 5. [5 pts] Use the hypothetical judgements in Question 4 and state the two structural properties weakening and contraction.

Question 6. [5 pts] Write a proposition that is logically equivalent to $(A \vee B) \supset C$. Your answer should not express trivial logical equivalence based on commutativity, idempotence, or properties of truth and falsehood. Here are examples of bad answers: $(B \vee A) \supset C$, $(A \vee (\top \wedge B)) \supset C$, and $(A \vee (\bot \vee B)) \supset C$.

Question 7. [5 pts] Suppose that your answer to the previous question is a proposition D. Give a proof of $((A \vee B) \supset C) \supset D\ \mathit{true}$ in the natural deduction style.

제 3 절 First-order logic [25pts]

Question 1. [5 pts] Write the introduction and elimination rules for the existential quantification using truth judgments of the form $C\ true$.

Question 2. [5 pts] Use the inference rules in Question 1 and show that local soundness holds for the existential quantification.

Question 3. [5 pts] Give the typing rules for proof terms for the existential quantification.

Question 4. [10 pts] Assume the following typing rules for characterizing natural numbers:

$$\frac{}{\mathsf{Nat_0} : Nat(\mathbf{0})}\ Zero \qquad \frac{}{\mathsf{Nat_s} : \forall x.Nat(x) \supset Nat(\mathbf{s}(x))}\ Succ$$

$$\frac{}{\mathsf{Eq_i} : \forall x.Eq(x,x)}\ Eq_i \qquad \frac{}{\mathsf{Eq_t} : \forall x.\forall y.\forall z.(Eq(x,y) \wedge Eq(x,z)) \supset Eq(y,z)}\ Eq_t$$

$$\frac{}{\mathsf{Lt_s} : \forall x.Lt(x,\mathbf{s}(x))}\ Lt_s \qquad \frac{}{\mathsf{Lt_\neg} : \forall x.\forall y.Eq(x,y) \supset \neg Lt(x,y)}\ Lt_\neg$$

Give a proof term of type $\neg\exists x.Eq(x,\mathbf{0}) \wedge Eq(x,\mathbf{s}(\mathbf{0}))$.

제 4 절 Datatypes [40pts]

Question 1. [5 pts] The introduction rules for datatype nat are:

$$\frac{}{\mathbf{0} \in \mathsf{nat}}\ \mathsf{natI}_0 \qquad \frac{t \in \mathsf{nat}}{\mathbf{s}(t) \in \mathsf{nat}}\ \mathsf{natI}_\mathsf{s}$$

We use $\mathsf{ind}\ u(t)\ \mathsf{of}\ u(\mathbf{0}) \Rightarrow M \mid u(\mathbf{s}(x)) \Rightarrow N$ for the proof term corresponding to the elimination rule based on induction on terms. Give its typing rule.

Question 2. [10 pts] We abbreviate $EQ(m, n)$ as $m =_\mathsf{N} n$ where both m and n belong to datatype nat. We use the following proof terms for the introduction rules for $m =_\mathsf{N} n$.

$$\frac{}{\mathsf{eqI}_0 : \mathbf{0} =_\mathsf{N} \mathbf{0}}\ =_\mathsf{N}\mathsf{I}_0 \qquad \frac{M : m =_\mathsf{N} n}{\mathsf{eqI}_\mathsf{s}(M) : \mathbf{s}(m) =_\mathsf{N} \mathbf{s}(n)}\ =_\mathsf{N}\mathsf{I}_\mathsf{s}$$

Design three proof terms for the elimination rules for $m =_\mathsf{N} n$ which are obtained by inverting the above two introduction rules, and give their typing rules. (Writing only the typing rules is okay.)

Question 3. **[5 pts]** Give a proof term $eqNat$ of type $\forall x \in \mathsf{nat}. x =_{\mathsf{N}} x$.

Question 4. [10 pts] Give a proof term $trans$ of type $\forall x \in \mathsf{nat}. \forall y \in \mathsf{nat}. \forall z \in \mathsf{nat}. x =_{\mathsf{N}} y \supset y =_{\mathsf{N}} z \supset x =_{\mathsf{N}} z$.

Question 5. [10 pts] Assume the following definition of $plus$:

$$plus \in \mathsf{nat} \to \mathsf{nat} \to \mathsf{nat}$$
$$plus \; = \; \lambda x \in \mathsf{nat}.\, \lambda y \in \mathsf{nat}.\, \mathbf{rec}\; p(x)\; \mathbf{of}\; p(\mathbf{0}) \Rightarrow y \mid p(\mathbf{s}(z)) \Rightarrow \mathbf{s}(p(z))$$

Using definitional equality, give a proof term $comp$ of type $\forall x \in \mathsf{nat}. \forall y \in \mathsf{nat}. x + \mathbf{s}(y) =_{\mathsf{N}} \mathbf{s}(x + y)$. You may use $eqNat$ given in Question 3.