

Name:

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## CSE-433 Logic in Computer Science 2007 Final exam — Sample Solution

- This is a closed-book exam. No other material is permitted.
- It consists of 4 problems worth a total of 175 points.
- There are 12 pages in this exam, including 3 work sheets.
- Try to use work sheets before writing your answers. Write your answers clearly and legibly.
- You have 3 hours for this exam.

	Problem 1	Problem 2	Problem 3	Problem 4	Total
Score					
Max	30	55	50	40	175

# 1 Admissibility of Harrop's rule [30 pts]

In constructive logic, there are some admissible rules that are not derivable. In the midterm exam, we considered Harrop's rule as an example of such a rule and showed that it is not derivable. As we said before, in this problem, we will show that Harrop's rule is indeed admissible. As a reminder, Harrop's rule is as follows:

$$\frac{\neg A \supset (B \vee C) \text{ true}}{(\neg A \supset B) \vee (\neg A \supset C) \text{ true}} \text{ Harrop}$$

where  $A, B$ , and  $C$  are all propositions.

You will need to use the following inference rules for neutral and normal judgments.

$$\begin{array}{c} \overline{A} \downarrow^x \\ \vdots \\ B \uparrow \\ \hline A \supset B \uparrow \end{array} \supset I \uparrow^x \quad \frac{A \supset B \downarrow \quad A \uparrow}{B \downarrow} \supset E \downarrow \quad \frac{A \uparrow \quad B \uparrow}{A \wedge B \uparrow} \wedge I \uparrow \quad \frac{A \wedge B \downarrow}{A \downarrow} \wedge E_{L \downarrow} \quad \frac{A \wedge B \downarrow}{B \downarrow} \wedge E_{R \downarrow}$$

$$\frac{A \uparrow}{A \vee B \uparrow} \vee I_{L \uparrow} \quad \frac{B \uparrow}{A \vee B \uparrow} \vee I_{R \uparrow} \quad \frac{A \vee B \downarrow \quad \begin{array}{c} \overline{A} \downarrow^x \quad \overline{B} \downarrow^y \\ \vdots \quad \vdots \\ C \uparrow \quad C \uparrow \end{array}}{C \uparrow} \vee E_{\uparrow}^{x,y}$$

$$\overline{\top} \uparrow \quad \top I \uparrow \quad \frac{\perp \downarrow}{C \uparrow} \perp E \uparrow \quad \frac{A \downarrow}{A \uparrow} \downarrow \quad \frac{\perp \uparrow}{\neg A \uparrow} \neg I \uparrow^x \quad \frac{\neg A \downarrow \quad A \uparrow}{\perp \downarrow} \neg E \downarrow$$

- You should annotate every part of your derivation with the name of the inference rule (e.g.,  $\supset E \downarrow$ ) and also a label if applicable (e.g.,  $\supset I \uparrow^x$ ). In particular, you should annotate each hypothesis with some variable (e.g.,  $\overline{A} \downarrow^x$ ).

Since we cannot directly show that Harrop's rule is admissible, we need to analyze the proof structure of the premise,  $\neg A \supset (B \vee C) \text{ true}$ , to show that  $(\neg A \supset B) \vee (\neg A \supset C) \text{ true}$  is provable. Recall that in constructive logic,  $A \text{ true}$  is provable if and only if  $A \uparrow$  is provable. Therefore we will show that Harrop's rule is admissible by showing that  $(\neg A \supset B) \vee (\neg A \supset C) \uparrow$  is provable in the presence of a proof of  $\neg A \supset (B \vee C) \uparrow$ .

*Proof.* The proof of  $\neg A \supset (B \vee C) \uparrow$ , if any, must be of the following form:

$$\frac{\mathcal{D} \left\{ \begin{array}{c} \overline{\neg A} \downarrow^x \\ \vdots \\ B \vee C \uparrow \end{array} \right.}{\neg A \supset (B \vee C) \uparrow} \supset I \uparrow^x$$

We need to show that  $(\neg A \supset B) \vee (\neg A \supset C) \uparrow$  is provable by exploring *all* possible cases of the structure of  $\mathcal{D}$ .

Case 1. Suppose that in  $\mathcal{D}$ , the rule  $\neg E_{\downarrow}$  is applied to  $\overline{\neg A \downarrow}^x$ . Then there must be a proof of  $A \uparrow$  under the hypothesis  $\overline{\neg A \downarrow}^x$ . Let this proof be  $\mathcal{E}$ . Use  $\mathcal{E}$  to build a proof of  $(\neg A \supset B) \vee (\neg A \supset C) \uparrow$  (10 pts):

$$\frac{\frac{\overline{\neg A \downarrow}^x}{\neg A \downarrow} \quad \left. \begin{array}{c} \vdots \\ A \uparrow \end{array} \right\} \mathcal{E}}{\neg E_{\downarrow}} \quad \frac{\frac{\perp \downarrow}{B \uparrow} \quad \perp E_{\uparrow}}{\neg A \supset B \uparrow} \supset I_{\uparrow}^x}{(\neg A \supset B) \vee (\neg A \supset C) \uparrow} \vee I_{L\uparrow}$$

Case 2. Suppose that in  $\mathcal{D}$ , the rule  $\vee I_{L\uparrow}$  is applied to deduce  $B \vee C \uparrow$ . Then there must be a proof of  $B \uparrow$  under the hypothesis  $\overline{\neg A \downarrow}^x$ . Let this proof be  $\mathcal{E}$ . Use  $\mathcal{E}$  to build a proof of  $(\neg A \supset B) \vee (\neg A \supset C) \uparrow$  (10 pts):

$$\frac{\mathcal{E} \left\{ \begin{array}{c} \overline{\neg A \downarrow}^x \\ \vdots \\ B \uparrow \end{array} \right.}{\neg A \supset B \uparrow} \supset I_{\uparrow}^x}{(\neg A \supset B) \vee (\neg A \supset C) \uparrow} \vee I_{L\uparrow}$$

Case 3. Suppose that in  $\mathcal{D}$ , the rule  $\vee I_{R\uparrow}$  is applied to deduce  $B \vee C \uparrow$ . Then there must be a proof of  $C \uparrow$  under the hypothesis  $\overline{\neg A \downarrow}^x$ . Let this proof be  $\mathcal{E}$ . Use  $\mathcal{E}$  to build a proof of  $(\neg A \supset B) \vee (\neg A \supset C) \uparrow$  (10 pts):

$$\frac{\mathcal{E} \left\{ \begin{array}{c} \overline{\neg A \downarrow}^x \\ \vdots \\ C \uparrow \end{array} \right.}{\neg A \supset C \uparrow} \supset I_{\uparrow}^x}{(\neg A \supset B) \vee (\neg A \supset C) \uparrow} \vee I_{R\uparrow}$$

The above three cases are the only possible cases of the structure of  $\mathcal{D}$ . Therefore in the presence of a proof of  $\neg A \supset (B \vee C) \uparrow$ , we can always prove  $(\neg A \supset B) \vee (\neg A \supset C) \uparrow$ , which means that Harrop's rule is admissible.

## 2 Datatypes [55 pts]

We use the following definition of proof terms for the predicate  $m < n$ :

proof term  $M ::= \dots \mid \text{ltl}_0 \mid \text{ltl}_s(M) \mid \text{ltE}_0(M) \mid \text{ltE}_s(M)$

$$\frac{}{\text{ltl}_0 : \mathbf{0} < \mathbf{s}(n)} < \text{l}_0 \quad \frac{M : m < n}{\text{ltl}_s(M) : \mathbf{s}(m) < \mathbf{s}(n)} < \text{l}_s \quad \frac{M : m < \mathbf{0}}{\text{ltE}_0(M) : C} < \text{E}_0 \quad \frac{M : \mathbf{s}(m) < \mathbf{s}(n)}{\text{ltE}_s(M) : m < n} < \text{E}_s$$

We use the following definition of proof terms for the predicate  $m =_{\mathbf{N}} n$ :

proof term  $M ::= \dots \mid \text{eqI}_0 \mid \text{eqI}_s(M) \mid \text{eqE}_{0s}(M) \mid \text{eqE}_{s0}(M) \mid \text{eqE}_s(M)$

$$\frac{}{\text{eqI}_0 : \mathbf{0} =_{\mathbf{N}} \mathbf{0}} =_{\mathbf{N}} \text{l}_0 \quad \frac{M : m =_{\mathbf{N}} n}{\text{eqI}_s(M) : \mathbf{s}(m) =_{\mathbf{N}} \mathbf{s}(n)} =_{\mathbf{N}} \text{l}_s$$

$$\frac{M : \mathbf{0} =_{\mathbf{N}} \mathbf{s}(n)}{\text{eqE}_{0s}(M) : C} =_{\mathbf{N}} \text{E}_{0s} \quad \frac{M : \mathbf{s}(m) =_{\mathbf{N}} \mathbf{0}}{\text{eqE}_{s0}(M) : C} =_{\mathbf{N}} \text{E}_{s0} \quad \frac{M : \mathbf{s}(m) =_{\mathbf{N}} \mathbf{s}(n)}{\text{eqE}_s(M) : m =_{\mathbf{N}} n} =_{\mathbf{N}} \text{E}_s$$

The following proof term represents a proof by induction on natural numbers:

proof term  $M ::= \dots \mid \text{ind } u(t) \text{ of } u(\mathbf{0}) \Rightarrow M \mid u(\mathbf{s}(x)) \Rightarrow N$

**Question 1. [5 pts]** Define a function *append* concatenating two lists:

$$\begin{aligned} \text{append } \mathbf{nil}^{\tau} t &= t \\ \text{append } (x :: l) t &= x :: (\text{append } l t) \end{aligned}$$

$\text{append} \in \text{list } \tau \rightarrow \text{list } \tau \rightarrow \text{list } \tau$

$\text{append} = \lambda y \in \text{list } \tau. \lambda z \in \text{list } \tau. \text{rec } f(y) \text{ of } f(\mathbf{nil}) \Rightarrow z \mid f(x :: l) \Rightarrow x :: f(l)$

**Question 2. [5 pts]** Give a proof term of type  $(\exists x \in \tau. A(x) \vee B(x)) \supset ((\exists x \in \tau. A(x)) \vee (\exists x \in \tau. B(x)))$ . You may omit type annotations in injection terms. For example, you may write  $\text{inl } M$  when  $\text{inl}_A M$  is expected.

$$\lambda z : \exists x \in \tau. A(x) \vee B(x). \text{let } \langle x, w \rangle = z \text{ in case } w \text{ of inl } y_1. \text{inl}_{\exists x \in \tau. B(x)} \langle x, y_1 \rangle \mid \text{inr } y_2. \text{inr}_{\exists x \in \tau. A(x)} \langle x, y_2 \rangle.$$

**Question 3. [5 pts]** The specification of a proof term *pred* of type  $\forall x \in \text{nat}. \neg(x =_{\mathbf{N}} \mathbf{0}) \supset \exists y \in \text{nat}. \mathbf{s}(y) =_{\mathbf{N}} x$  is:

$$\begin{aligned} \text{pred } \mathbf{0} \quad v : \neg(x =_{\mathbf{N}} \mathbf{0}) &= \text{proof term of type } \exists y \in \text{nat}. \mathbf{s}(y) =_{\mathbf{N}} x \\ \text{pred } \mathbf{0} \quad v : \neg(\mathbf{0} =_{\mathbf{N}} \mathbf{0}) &= \text{abort}_{\exists y \in \text{nat}. \mathbf{s}(y) =_{\mathbf{N}} \mathbf{0}} (v \text{ eqI}_0) \\ \text{pred } \mathbf{s}(x') \quad v : \neg(\mathbf{s}(x') =_{\mathbf{N}} \mathbf{0}) &= \langle x', \text{eqI}_s(\text{eqNat } x') \rangle \end{aligned}$$

Here *eqNat* is defined as  $\lambda x \in \text{nat}. \text{ind } u(x) \text{ of } u(\mathbf{0}) \Rightarrow \text{eqI}_0 \mid u(\mathbf{s}(x')) \Rightarrow \text{eqI}_s(u(x'))$ . Give a definition of *pred* based on this specification.

$$\text{pred} = \lambda x \in \text{nat}. \text{case } x \text{ of } \begin{cases} \mathbf{0} \Rightarrow \lambda v : \neg(\mathbf{0} =_{\mathbf{N}} \mathbf{0}). \text{abort}_{\exists y \in \text{nat}. \mathbf{s}(y) =_{\mathbf{N}} \mathbf{0}} (v \text{ eqI}_0) \\ \mathbf{s}(x') \Rightarrow \lambda v : \neg(\mathbf{s}(x') =_{\mathbf{N}} \mathbf{0}). \langle x', \text{eqI}_s(\text{eqNat } x') \rangle \end{cases}$$

**Question 4. [15 pts]** We wish to find a proof term *ltSucc* of type

$$\forall m \in \text{nat}. \forall n \in \text{nat}. m < \mathbf{s}(n) \supset (m =_{\mathbf{N}} n \vee m < n)$$

(which you used in Assignment 8). Write a specification of  $ltSucc$  (similarly to  $pred$ ) and then translate it to a definition as in Question 3.

Specification:

$$\begin{array}{llll}
m & n & v : m < \mathbf{s}(n) & \text{proof term of type } m =_{\mathbf{N}} n \vee m < n \\
ltSucc \ \mathbf{0} & \mathbf{0} & v : \mathbf{0} < \mathbf{s}(\mathbf{0}) & = \text{inl}_{\mathbf{0} < \mathbf{0}} \text{eq}_{\mathbf{0}} \\
ltSucc \ \mathbf{0} & \mathbf{s}(n') & v : \mathbf{0} < \mathbf{s}(\mathbf{s}(n')) & = \text{inr}_{\mathbf{0} =_{\mathbf{N}} \mathbf{s}(n')} \text{ltl}_{\mathbf{0}} \\
ltSucc \ \mathbf{s}(m') & \mathbf{0} & v : \mathbf{s}(m') < \mathbf{s}(\mathbf{0}) & = \text{ltE}_{\mathbf{0}}(\text{ltE}_{\mathbf{s}}(v)) \\
ltSucc \ \mathbf{s}(m') & \mathbf{s}(n') & v : \mathbf{s}(m') < \mathbf{s}(\mathbf{s}(n')) & = \text{case } ltSucc \ m' \ n' \ \text{ltE}_{\mathbf{s}}(v) \text{ of } \left\{ \begin{array}{l} \text{inl } x. \text{inl}_{\mathbf{s}(m') < \mathbf{s}(n')} \text{eq}_{\mathbf{s}}(x) \\ \text{inr } y. \text{inr}_{\mathbf{s}(m') =_{\mathbf{N}} \mathbf{s}(n')} \text{ltl}_{\mathbf{s}}(y) \end{array} \right.
\end{array}$$

Definition:

$$\lambda m \in \text{nat. ind } u(m) \text{ of } \left\{ \begin{array}{l} u(\mathbf{0}) \Rightarrow \lambda n \in \text{nat. case } n \text{ of } \left\{ \begin{array}{l} \mathbf{0} \Rightarrow \lambda v : \mathbf{0} < \mathbf{s}(\mathbf{0}). \text{inl}_{\mathbf{0} < \mathbf{0}} \text{eq}_{\mathbf{0}} \\ \mathbf{s}(n') \Rightarrow \lambda v : \mathbf{0} < \mathbf{s}(\mathbf{s}(n')). \text{inr}_{\mathbf{0} =_{\mathbf{N}} \mathbf{s}(n')} \text{ltl}_{\mathbf{0}} \end{array} \right. \\ u(\mathbf{s}(m')) \Rightarrow \lambda n \in \text{nat. case } n \text{ of } \left\{ \begin{array}{l} \mathbf{0} \Rightarrow \lambda v : \mathbf{s}(m') < \mathbf{s}(\mathbf{0}). \text{ltE}_{\mathbf{0}}(\text{ltE}_{\mathbf{s}}(v)) \\ \mathbf{s}(n') \Rightarrow \lambda v : \mathbf{s}(m') < \mathbf{s}(\mathbf{s}(n')). \\ \text{case } u(m') \ n' \ \text{ltE}_{\mathbf{s}}(v) \text{ of } \left\{ \begin{array}{l} \text{inl } x. \text{inl}_{\mathbf{s}(m') < \mathbf{s}(n')} \text{eq}_{\mathbf{s}}(x) \\ \text{inr } y. \text{inr}_{\mathbf{s}(m') =_{\mathbf{N}} \mathbf{s}(n')} \text{ltl}_{\mathbf{s}}(y) \end{array} \right. \end{array} \right.
\end{array}$$

**Question 5. [5 pts]** Write the elimination rule for the predicate  $m =_{\mathbf{N}} n$  based on induction on predicates (which is given in the Course Notes):

$$\frac{\overline{m \in \text{nat}} \quad \overline{n \in \text{nat}} \quad \overline{m =_{\mathbf{N}} n}^w \quad \overline{A(m, n) \text{ true}}^{u(m, n)} \quad \vdots \quad \overline{A(\mathbf{0}, \mathbf{0}) \text{ true}} \quad \overline{A(\mathbf{s}(m), \mathbf{s}(n)) \text{ true}}}{\overline{m_0 =_{\mathbf{N}} n_0 \text{ true}} \quad \overline{A(m_0, n_0) \text{ true}}} =_{\mathbf{N}} E_I^{w, u(m, n)}$$

**Question 6. [5 pts]** Use this elimination rule to prove  $\forall x \in \text{nat.} \forall y \in \text{nat.} x =_{\mathbf{N}} y \supset y =_{\mathbf{N}} x \text{ true}$ :

$$\frac{\overline{x =_{\mathbf{N}} y \text{ true}}^w \quad \overline{\mathbf{0} =_{\mathbf{N}} \mathbf{0} \text{ true}} =_{\mathbf{N}} \text{!}_{\mathbf{0}} \quad \overline{n =_{\mathbf{N}} m \text{ true}}^{u(m, n)} \quad \overline{\mathbf{s}(n) =_{\mathbf{N}} \mathbf{s}(m) \text{ true}} =_{\mathbf{N}} \text{!}_{\mathbf{s}}}{\overline{y =_{\mathbf{N}} x \text{ true}} \quad \overline{x =_{\mathbf{N}} y \supset y =_{\mathbf{N}} x \text{ true}} \supset \text{!}^w} =_{\mathbf{N}} E_I^{w, u(m, n)}$$

$$\frac{\overline{\forall y \in \text{nat.} x =_{\mathbf{N}} y \supset y =_{\mathbf{N}} x \text{ true}} \forall \text{!}}{\overline{\forall x \in \text{nat.} \forall y \in \text{nat.} x =_{\mathbf{N}} y \supset y =_{\mathbf{N}} x \text{ true}} \forall \text{!}}$$

**Question 7. [5 pts]** Assuming definitional equality, give a proof term of type  $\forall x \in \text{nat.} \forall y \in \text{nat.} x + \mathbf{s}(y) =_{\mathbf{N}} \mathbf{s}(x + y)$ :

$$\lambda x \in \text{nat.} \lambda y \in \text{nat.} \text{ind } u(x) \text{ of } \left\{ \begin{array}{l} u(\mathbf{0}) \Rightarrow \text{eqNat } \mathbf{s}(y) \\ u(\mathbf{s}(x')) \Rightarrow \text{eq}_{\mathbf{s}}(u(x')) \end{array} \right.$$

**Question 8. [5 pts]** The following inference rule is derivable. True or false?

$$\frac{A(t) \text{ true} \quad t =_{\mathbf{N}} s}{A(s) \text{ true}} \text{NatEqE}$$

False

**Question 9. [5 pts]** The rule  $\text{NatEqE}$  in the previous question is admissible. True or false?

True



$$\text{Case } \frac{\Gamma; \Delta \vdash_{\mathcal{K}} M : A \supset B \quad \Gamma; \Delta \vdash_{\mathcal{K}} N : A}{\Gamma; \Delta \vdash_{\mathcal{K}} M N : B} \supset E$$

$$(M N)^{\circ} = \lambda k : B^{\circ} \supset \perp. M^{\circ} (\lambda x : A^{\circ} \supset \neg \neg B^{\circ}. N^{\circ} (\lambda y : A^{\circ}. x y k))$$

$$\text{Case } \frac{\Gamma; \Delta, x : A \text{ false} \vdash_{\mathcal{K}} M : A}{\Gamma; \Delta \vdash_{\mathcal{K}} \text{callcc } x : A \text{ false}. M : A} \text{Callcc}$$

$$(\text{callcc } x : A \text{ false}. M)^{\circ} = \lambda k : A^{\circ} \supset \perp. [k/x]M^{\circ} k$$

$$\text{Case } \frac{\Gamma; \Delta \vdash_{\mathcal{K}} M : A \quad x : A \text{ false} \in \Delta}{\Gamma; \Delta \vdash_{\mathcal{K}} \text{throw } M \text{ to } x : C} \text{Throw}$$

$$(\text{throw } M \text{ to } x)^{\circ} = \lambda k : C^{\circ} \supset \perp. M^{\circ} x$$

**Question 3. [25 pts]** The above CPS transformation corresponds to the call-by-value reduction strategy. To be specific, the translation of  $M N$  specifies that we finish evaluating  $N$  before we apply the function from  $M$  to the result of evaluating  $N$ .

In this question, we will develop a variant of the CPS translation that corresponds to the call-by-name reduction strategy. We use the following double-negation translation (known as *Kolmogorov double negation translation*) in which  $(A \supset B)^{\circ}$  places  $\neg \neg$  before both  $A^{\circ}$  and  $B^{\circ}$ :

$$\begin{aligned} P^{\circ} &= P \\ (A \supset B)^{\circ} &= \neg \neg A^{\circ} \supset \neg \neg B^{\circ} \end{aligned}$$

For each case below, complete the CPS translation  $M^{\circ}$  of a given proof term  $M$ . Your translation should satisfy the invariant specified in the following theorem:

**Theorem.** *If  $\Gamma; \Delta \vdash_{\mathcal{K}} M : C$ , there exists a proof term  $M^{\circ}$  such that  $\Gamma^{\circ}, \neg \Delta^{\circ} \vdash_{\mathcal{I}} M^{\circ} : \neg \neg C^{\circ}$  where*

$$\begin{aligned} \Gamma^{\circ} &= \{x : \neg \neg A^{\circ} \mid x : A \in \Gamma\} \\ \neg \Delta^{\circ} &= \{x : \neg A^{\circ} \mid x : A \text{ false} \in \Delta\}. \end{aligned}$$

Note the change in the definition of  $\Gamma^{\circ}$  which now assigns to  $x$  type  $\neg \neg A^{\circ}$ . The translation of  $\text{callcc } x : A \text{ false}. M$  and  $\text{throw } M \text{ to } x$  is the same as in the previous CPS translation and is omitted.

$$\text{Case } \frac{x : A \in \Gamma}{\Gamma; \Delta \vdash_{\mathcal{K}} x : A} \text{Hyp}$$

$$x^\circ = \lambda k : A^\circ \supset \perp. x k$$

$$\text{Case } \frac{\Gamma, x : A; \Delta \vdash_{\kappa} M : B}{\Gamma; \Delta \vdash_{\kappa} \lambda x : A. M : A \supset B} \supset I$$

$$(\lambda x : A. M)^\circ = \lambda k : (\neg\neg A^\circ \supset \neg\neg B^\circ) \supset \perp. k (\lambda x : \neg\neg A^\circ. M^\circ)$$

$$\text{Case } \frac{\Gamma; \Delta \vdash_{\kappa} M : A \supset B \quad \Gamma; \Delta \vdash_{\kappa} N : A}{\Gamma; \Delta \vdash_{\kappa} M N : B} \supset E$$

$$(M N)^\circ = \lambda k : B^\circ \supset \perp. M^\circ (\lambda x : \neg\neg A^\circ \supset \neg\neg B^\circ. x N^\circ k)$$

## 4 Linear logic [40 pts]

For each proposition  $A$  in linear logic below, prove its truth using natural deduction with the linear hypothetical judgment  $\Delta \vdash A$ . You should annotate every part of your derivation with the name of the inference rule. If its truth is unprovable, state so.

**Question 1. [5 pts]**  $(A \otimes \mathbf{1}) \multimap A$

$$\frac{\frac{\frac{}{A \otimes \mathbf{1} \vdash A \otimes \mathbf{1}}{\text{Hyp}} \quad \frac{\frac{\frac{}{\mathbf{1} \vdash \mathbf{1}}{\text{Hyp}} \quad \frac{\frac{}{A \vdash A}}{\text{Hyp}}}{A, \mathbf{1} \vdash A} \otimes E}}{A \otimes \mathbf{1} \vdash A} \otimes E}{\vdash (A \otimes \mathbf{1}) \multimap A} \multimap I$$

**Question 2. [5 pts]**  $(A \oplus \mathbf{0}) \multimap A$

$$\frac{\frac{\frac{}{A \oplus \mathbf{0} \vdash A \oplus \mathbf{0}}{\text{Hyp}} \quad \frac{\frac{}{A \vdash A}}{\text{Hyp}} \quad \frac{\frac{}{\mathbf{0} \vdash \mathbf{0}}{\text{Hyp}}}{\mathbf{0} \vdash A} \oplus E}}{A \oplus \mathbf{0} \vdash A} \oplus E}{\vdash (A \oplus \mathbf{0}) \multimap A} \multimap I$$

**Question 3. [5 pts]**  $(A \& A) \multimap A$

$$\frac{\frac{\frac{}{A \& A \vdash A \& A}}{\text{Hyp}} \quad \frac{}{A \& A \vdash A} \& E_L}{\vdash A \& A \multimap A} \multimap I$$

**Question 4. [5 pts]**  $(A \oplus A) \multimap A$

$$\frac{\frac{\frac{}{A \oplus A \vdash A \oplus A}}{\text{Hyp}} \quad \frac{\frac{}{A \vdash A}}{\text{Hyp}} \quad \frac{\frac{}{A \vdash A}}{\text{Hyp}}}{A \oplus A \vdash A} \oplus E}{\vdash A \oplus A \multimap A} \multimap I$$

**Question 5. [5 pts]**  $(A \oplus (B \otimes C)) \multimap ((A \oplus B) \otimes (A \oplus C))$  *true* is provable. True or false?  
False.

**Question 6. [5 pts]**  $(A \oplus (B \& C)) \multimap ((A \oplus B) \& (A \oplus C))$  *true* is provable. True or false?  
True.

**Question 7. [5 pts]**  $(!A \otimes !B) \multimap (A \& B)$  *true* is provable. True or false?  
True.

**Question 8. [5 pts]**  $((A \multimap !A) \& (B \multimap !B)) \multimap ((A \otimes B) \multimap !(A \otimes B))$  *true* is provable. True or false?  
False.

## Work sheet

## Work sheet

## Work sheet