CSE-433 Assignment - *Inductive Proofs*

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Due at class, Tuesday, September 6

Welcome to CSE-433 Logic in Computer Science! In this assignment, you will practice inductive proofs.

- For this assignment, you are free to discuss proof ideas and techniques with your classmates. There is no penalty for discussing the assignment with your classmates.
- Please write your proofs clearly and legibly.

1 Matched parentheses

Consider the inference rules for the judgment s mparen and s lparen for recognizing strings of matched parentheses:

$$\frac{}{\epsilon \text{ mparen}}\ Meps \qquad \frac{s \text{ mparen}}{(s) \text{ mparen}}\ Mpar \qquad \frac{s_1 \text{ mparen} \quad s_2 \text{ mparen}}{s_1\ s_2 \text{ mparen}}\ Mseq$$

$$\frac{}{\epsilon \text{ lparen}}\ Leps \qquad \frac{s_1 \text{ lparen} \quad s_2 \text{ lparen}}{(s_1)\ s_2 \text{ lparen}}\ Lseq$$

We use ϵ to denote an empty string (*i.e.*, $\epsilon s = s = s\epsilon$).

Theorem 1.1. *If* s lparen*, then* s mparen*.*

Question 1. Give a proof of Theorem 1.1.

Consider another inductive definition of strings of matched parentheses where we use a new judgment s tparen:

$$\frac{}{\epsilon \text{ tparen}}\ Teps \qquad \frac{s_1 \text{ tparen} \quad s_2 \text{ tparen}}{s_1\ (s_2) \text{ tparen}}\ Tseq$$

Lemma 1.2. *If* s tparen *and* s' tparen*, then* $s\ s'$ tparen*.*

Question 2. Give a proof of Lemma 1.2.

Theorem 1.3. *If* s mparen*, then* s tparen*.*

Question 3. Use Lemma 1.2 to prove Theorem 1.3.

Theorems 1.1 and 1.3 imply that s lparen implies s tparen. Now we wish to prove this result directly, *i.e.*, without using the judgment s mparen:

Theorem 1.4. *If* s lparen*, then* s tparen*.*

Question 4. Try to prove Theorem 1.4 by rule induction on s lparen. If you can complete the proof, write it. If you need a lemma to complete the proof, state the lemma, prove it, and use it to complete the proof of Theorem 1.4.