

Name:

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CSE-433 Logic in Computer Science 2011 Explicit weakening and contraction rules [100 pts]

Here is a sequent calculus for propositional logic in which both weakening and contraction are built into the system:

$$\begin{array}{c} \frac{}{\Gamma, A \longrightarrow A} \textit{Init} \quad \frac{\Gamma, A \wedge B, A \longrightarrow C}{\Gamma, A \wedge B \longrightarrow C} \wedge L_L \quad \frac{\Gamma, A \wedge B, B \longrightarrow C}{\Gamma, A \wedge B \longrightarrow C} \wedge L_R \quad \frac{\Gamma \longrightarrow A \quad \Gamma \longrightarrow B}{\Gamma \longrightarrow A \wedge B} \wedge R \\ \\ \frac{\Gamma, A \supset B \longrightarrow A \quad \Gamma, A \supset B, B \longrightarrow C}{\Gamma, A \supset B \longrightarrow C} \supset L \quad \frac{\Gamma, A \longrightarrow B}{\Gamma \longrightarrow A \supset B} \supset R \\ \\ \frac{\Gamma, A \vee B, A \longrightarrow C \quad \Gamma, A \vee B, B \longrightarrow C}{\Gamma, A \vee B \longrightarrow C} \vee L \quad \frac{\Gamma \longrightarrow A}{\Gamma \longrightarrow A \vee B} \vee R_L \quad \frac{\Gamma \longrightarrow B}{\Gamma \longrightarrow A \vee B} \vee R_R \\ \\ \frac{}{\Gamma \longrightarrow \top} \top R \quad \frac{}{\Gamma, \perp \longrightarrow C} \perp L \end{array}$$

We wish to design another equivalent sequent calculus which has explicit weakening and contraction rules. The new sequent calculus uses a new form of sequent $\Gamma \Longrightarrow A$. We introduce two new inference rules (for weakening and contraction):

$$\frac{\Gamma \Longrightarrow C}{\Gamma, A \Longrightarrow C} \textit{Weaken} \quad \frac{\Gamma, A, A \Longrightarrow C}{\Gamma, A \Longrightarrow C} \textit{Contraction}$$

Rewrite the following rules using the new sequent $\Gamma \Longrightarrow A$ so that $\Gamma \longrightarrow A$ implies $\Gamma \Longrightarrow A$ and vice versa. Make sure that weakening and contraction are *not* built into your rules.

Init:

$\wedge L_L$:

$\wedge R:$

$\wedge R:$

$\supset R:$

$\forall L:$

$\forall R_L:$

$\top R:$

$\perp L:$