# Electrowetting
## Microfluidic Applications and Electromechanical Theory


Kwan Hyoung Kang

Department of Mechanical Engineering
Pohang University of Science and Technology
khkang@postech.edu


Presented at Department of Chemical Engineering
POSTECH
2003. 9. 15

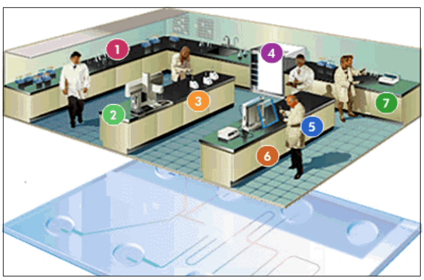
---

Chemistry + Mechanical Engineering

= "Chemical Engineering"

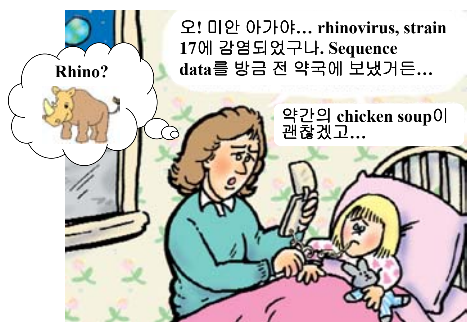
---

## Micro-Total Analysis System (μTAS) ≈ Lab-on-a-chip




---

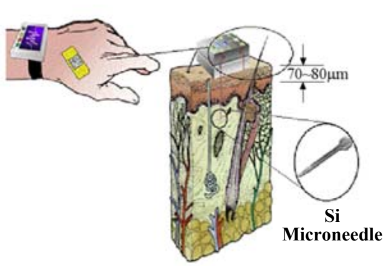
## Everyone's a (future) Chemist




(*from M. Burns 2002, Science* )

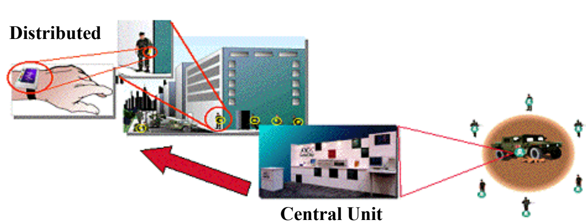
---

## Wrist-mounted chem/bio assay




---

## Wrist-mounted chem/bio assay

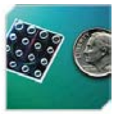


## Microfluidics

- **Microfluidics = Microhydrodynamics + Surface Chemistry + MEMS Tech.**

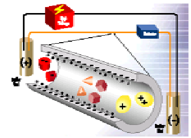
$$\frac{\text{surface force}}{\text{volume force}} \propto \frac{l^2}{l^3} = \frac{1}{l}$$

- **Central role in μTAS**
  - **Transport**
  - **Mixing**
  - **Separation**
  - **etc.**



## Liquid handling on sub-millimeter scales

- **Electroosmotic flow** – relatively high voltage ~ 1kV, slow ~ 0.1mm/s



- **Mechanical syringes and actuator** – complicated to fabricate, expensive

## Liquid handling on sub-millimeter scales

- **Creation of gradient in surface tension**
  - Photonic, Thermocapillary
  - **Electric: low cost, fast delivery, relatively easy to implement.**

## Surface tension in action


(National Geographic Explorer/MSNBC)

$$\frac{\text{surface force}}{\text{volume force}} \propto \frac{l^2}{l^3} = \frac{1}{l}$$

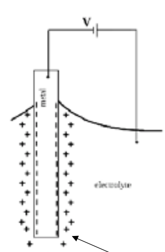
## Electrowetting control of liquid droplets

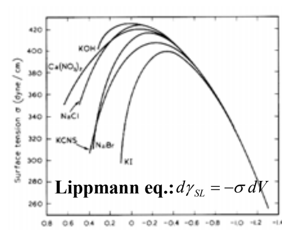

**Prof. C.-J. Kim (UCLA) (http://simony.seas.ucla.edu/)**


**Prof. R. B. Fair (Duke Univ.) (http://www.ece.duke.edu /Research/microfluidics/)**

## Effect of electrical charge on wetting

### Electrocapillarity and electrocapillary curve



# Gabriel Lippmann (1845~1921)

**French Experimental Physicist**

*One of the first color photo "Parrot" made by Lippmann in 1891.*

**Nobel Prize for Physics in 1908 for producing the first color photographic plate.**

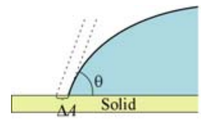
# Young's equation by energy method

- The change in surface free energy due to small displacement of the liquid

$$\Delta G^s = \Delta A(\gamma_{SL} - \gamma_{SV^o}) + \Delta A \gamma_{LV} \cos(\theta - \Delta\theta)$$

- At equilibrium

$$\lim_{\Delta A \to 0} \frac{\Delta G^s}{\Delta A} = 0 \longrightarrow (\gamma_{SL} - \gamma_{SV^o}) + \gamma_{LV} \cos\theta = 0$$



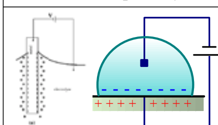
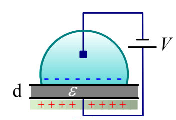

# Contact angle change by electrocapillarity

- Young's equation: $\gamma_{LV} \cos\theta = \gamma_{SV^o} - \gamma_{SL}$
- Change of surface tension due to charge

  Lippmann equation: $d\gamma_{SL} = -\sigma \, dV$
- Contact angle change

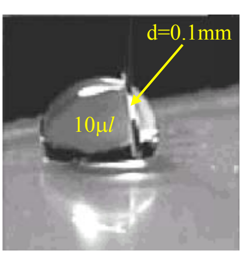
$$\cos\theta = \cos\theta_o - \frac{1}{\gamma_{LV}} \int \sigma \, dV; \qquad \cos\theta_o = \frac{\gamma_{SV^o} - \gamma_{SL}}{\gamma_{LV}}$$

# Electrocapillarity vs Electrowetting

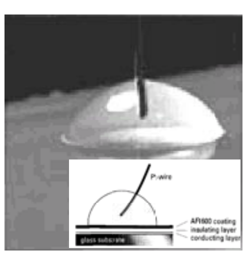
| | Electrocapillarity | Electrowetting |
|---|---|---|
| Classification |  |   |
| Surface tension | $d\gamma_{SL} = -\sigma \, dV$ | $d\gamma_{SL} = -\sigma \, dV$ |
| Surface charge | $\sigma \sim \varepsilon\kappa V$ | $\sigma = \varepsilon V / d$ |
| Contact angle | $\cos\theta = \cos\theta_o - \frac{1}{\gamma_{LV}} \int \sigma \, dV$ | $\cos\theta = \cos\theta_o + \frac{\varepsilon V^2}{2\gamma_{LV} d}$ Lippmann-Young eqn. |

# Contact angle control by electrowetting

**$10^{-4}$ M KNO3 droplet on Teflon AF1600/Parylene C surface**

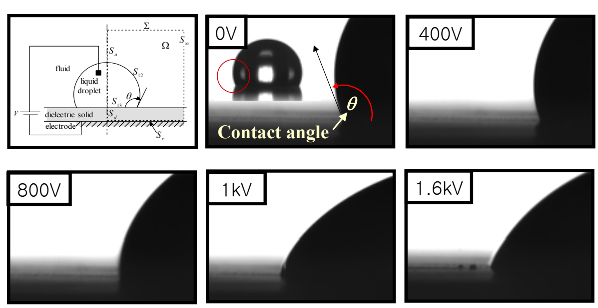



(a) 0V

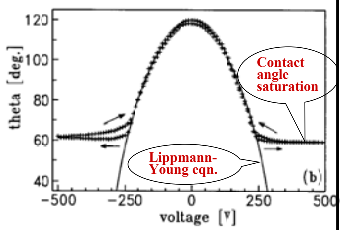



(a) 200V

# Contact angle control by electrowetting

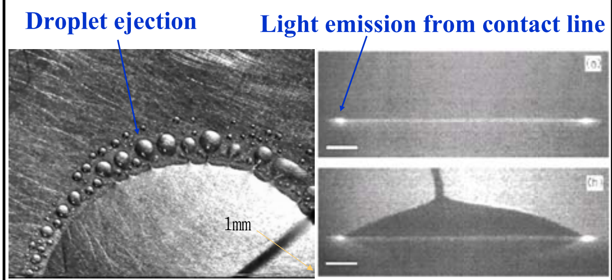


## Contact angle control by electrowetting




H. J. J. Verheijen *et al.* (*Philips Res. Lab. Eindhoven, The Netherlands*) (1999) *Langmuir 15*, 6616.

## Contact line instability (after saturation)




M. Vallet *et al. Eur. Phys. J. B*, 1999.

## Electrocapillarity vs Electrowetting

| Classification | Electrocapillarity | EW on Polymers | EW on SAMs |
|---|---|---|---|
| Capacitance | εκ, ~$10^4$ μF/m$^2$ | ε/*d*, ~1 μF/m$^2$ | ε/δ, ~$10^3$ μF/m$^2$ |
| Equiv. dielectric. Thickness | κ, ~ 1nm | *d*, 10μm | δ, ~ 10nm |
| Specific adsorption | Influential | Electro-chemically inert | Influential |
| Contact angle hysteresis | moderate | Depends on polymers | In general, significant |

## Electrocapillarity vs Electrowetting

| Classification | Electrocapillarity | EW on Polymers | EW on SAMs |
|---|---|---|---|
| Applied voltage | 0 ~ 5V | 0V~1kV | 0 ~ 5V |
| Reversibility | bad | good | bad |
| Dominant interaction force | Coulombic, Chemical, | Coulombic | Coulombic, Chemical |

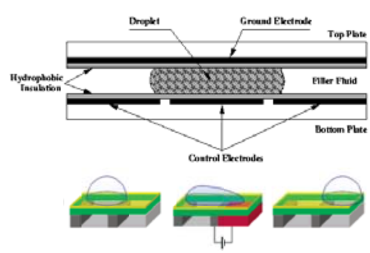
## Electrowetting control of liquid droplets

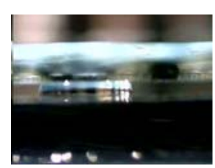

**Prof. C.-J. Kim (UCLA)**
**(http://simony.seas.ucla.edu/)**


**Prof. R. B. Fair (Duke Univ.)**
**(http://www.ece.duke.edu /Research/microfluidics/)**

## Actuation of microdroplet by electrowetting

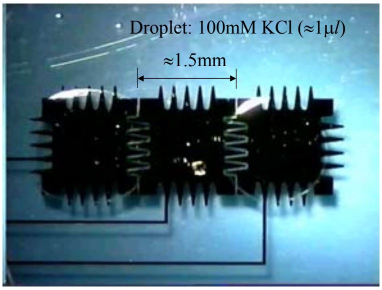
**"No need for pumps, valves, or even fixed channels. "**



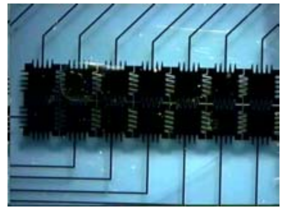



M.G. Pollack et al. (Duke U., Dept Elect. Eng.) (2000) Appl. Phys. Lett. 77, 1725.

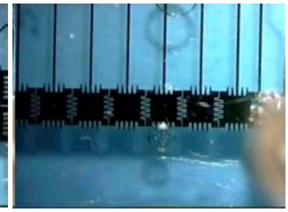
## Splitting of a droplet




## Other applications

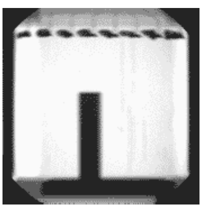


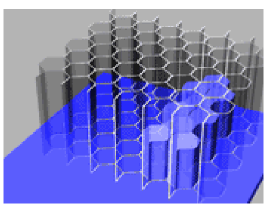
Flow of a droplet on a 2-D array



Dispensing of droplets

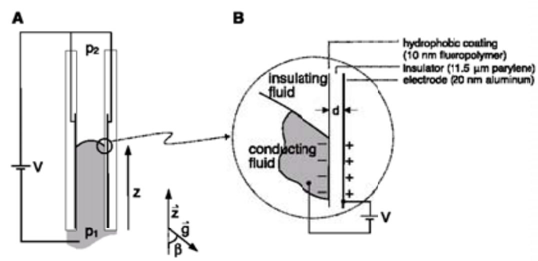
## Liquid actuation in microcapillary





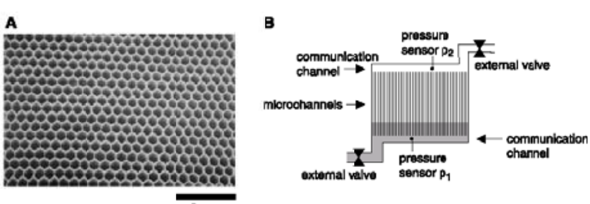
M. W. J. Prins,* W. J. J. Welters, J. W. Weekamp, "Fluid Control in Multichannel Structures by Electrocapillary Pressure," Science, January 12, 2001.

## Liquid actuation in microcapillary




M. W. J. Prins,* W. J. J. Welters, J. W. Weekamp, "Fluid Control in Multichannel Structures by Electrocapillary Pressure," Science, January 12, 2001.

## Liquid actuation in microcapillary

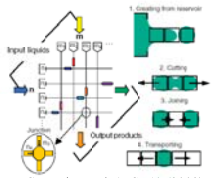



M. W. J. Prins,* W. J. J. Welters, J. W. Weekamp, "Fluid Control in Multichannel Structures by Electrocapillary Pressure," Science, January 12, 2001.

# Applications

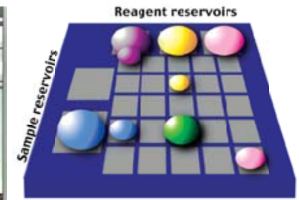
## Why electrowetting for lab-on-a-chip?

- Droplet-based microfluidic operations
  - By programmed electric signals rather than by complex physical structures.
  - Fabrication process becomes very simple.

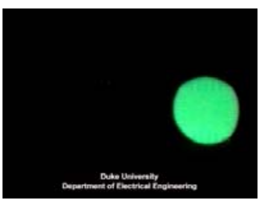

C.-J. Kim et al. (UCLA) (2002)

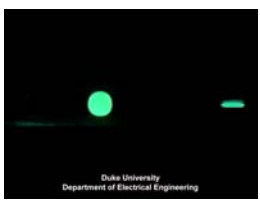
## Digital microfluidics by electrowetting

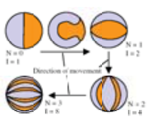




Other advantages of electrowetting
: fast liquid actuation, low power consumptions.

## Mixing of droplet by electrowetting

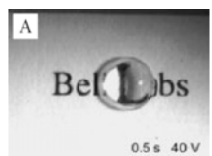


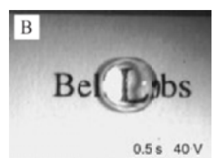



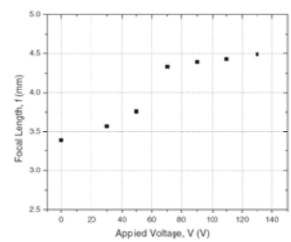




Prof. R. B. Fair (Duke Univ.)
(http://www.ece.duke.edu
/Research/microfluidics/)

## Optical lens (1)

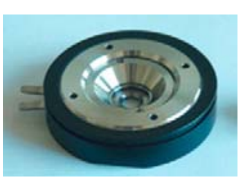


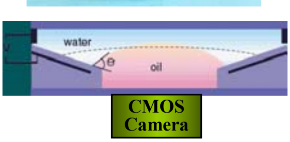




Yang et al. (Bell Lab.)
(2003)Adv. Mater. 15(11) 940.

## Optical lens (2)



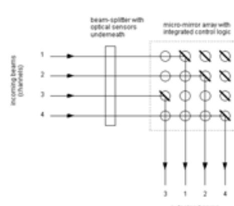


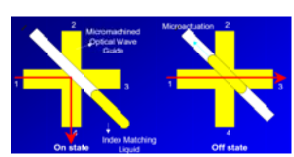

http://www.varioptic.com/

**Benefits**
- No moving parts, direct electric control
- Fast response (0.02s for a 5mm diameter)
- Very good optical quality
- Reduced electrical consumption

**Applications**
- Medical optics
- Autofocus lenses for CCD cameras
- Bar code readers

## Optical switch for optical network





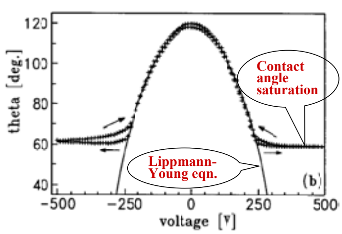
# Development of Electromechanical Theory

## Why electromechanical approach?

- Some MEs are unfamiliar with the energy method.

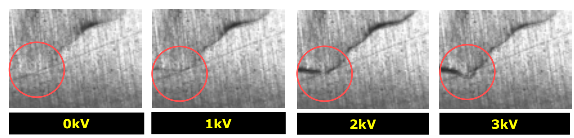
## Why electromechanical approach?

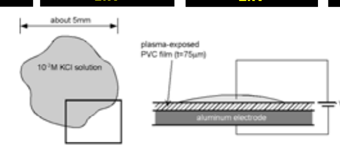
- Limited validity of Lippmann–Young equation due to contact-angle saturation.




## Why electromechanical approach?

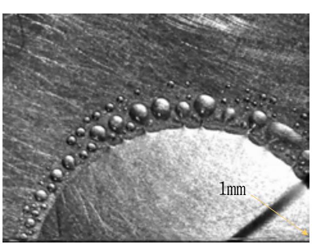
- Increase of contact angle by electrowetting?

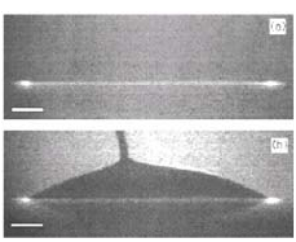



## Why electromechanical approach?

- Mechanism of the contact line instability?

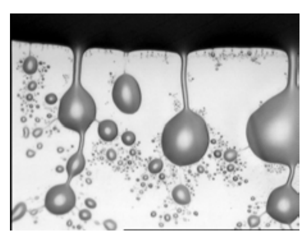


M. Vallet *et al. Eur. Phys. J.* B, 1999.

## Why electromechanical approach?

- Mechanism of the contact line instability?

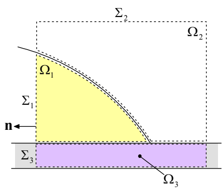


## Why electromechanical approach?

- Limited validity of Lippmann–Young equation due to contact-angle saturation.
- Mechanism of the contact line instability?
- An alternative approach is necessary to analyze the complex phenomena.

## Essence of the electromechanical theory

- Use of Gauss theorem and the Maxwell stress tensor.




$$\mathbf{F} = \int_\Omega \mathbf{f}\, dV = \int_\Sigma \mathbf{T}\cdot\mathbf{n}\, dS$$

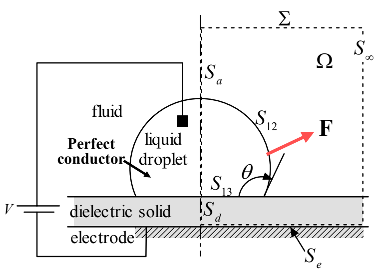
$$\mathbf{f} = \rho_f \mathbf{E} - \tfrac{1}{2}E^2\nabla\varepsilon \quad \text{(Korteweg–Helmholtz force density)}$$

$$\mathbf{T} = -\tfrac{1}{2}\varepsilon E^2 \mathbf{I} + \varepsilon \mathbf{E}\mathbf{E} \quad \text{(Maxwell stress tensor)}$$

$$W_{el} = -\mathbf{F}\cdot\mathbf{e}_x \quad \text{(wetting tension)}$$

$$\cos\theta = \cos\theta_o + \frac{W_{el}}{\gamma} \quad \text{(modified Young's equation)}$$

## Direct integration of Maxwell stress




$$\nabla^2\varphi = 0$$

$$\mathbf{F} = \int_{S_{12}+S_{13}} \mathbf{T}\cdot\mathbf{n}\, dS$$

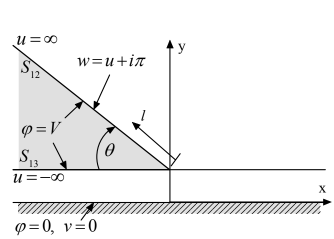
$$\mathbf{T} = -\tfrac{1}{2}\varepsilon E^2 \mathbf{I} + \varepsilon \mathbf{E}\mathbf{E}$$

Maxwell stress tensor

Note: Effect of electrical double layer is neglected.

## Analysis of potential problem

### Schwarz-Christoffel transformation




$$Z = \int_{i\pi}^{w} (e^{w'}+1)^\alpha\, dw' + i\pi$$

$$Z = x + iy$$

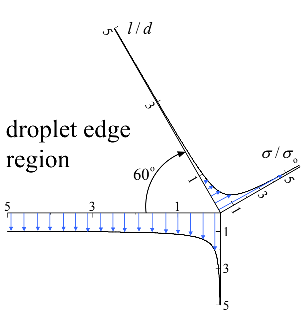
$$w = u + iv = u + i\left(\frac{\pi}{V}\right)\varphi$$

$$\mathbf{T}\cdot\mathbf{n} = (-\tfrac{1}{2}\varepsilon E^2\mathbf{I} + \varepsilon\mathbf{E}\mathbf{E})\cdot\mathbf{n}$$

$$= \frac{\varepsilon E^2}{2}\mathbf{n} = \frac{\sigma^2}{2\varepsilon}\mathbf{n}$$

Important assumption: $\varepsilon_{substrate} = \varepsilon_{air}$

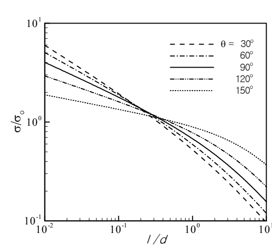
## Charge distribution at the surface



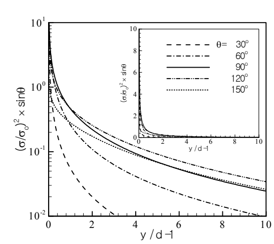

$$\sigma = \varepsilon\,|E_x - iE_y| = \frac{\varepsilon V}{e}\left|\frac{i}{dZ/dw}\right|$$

$$\frac{\sigma}{\sigma_o} = \begin{cases} 1/(e^u-1)^a, & \text{on } S_{12}, \\ 1/(1-e^u)^a, & \text{on } S_{13}. \end{cases}$$

## Maxwell stress acting on surface



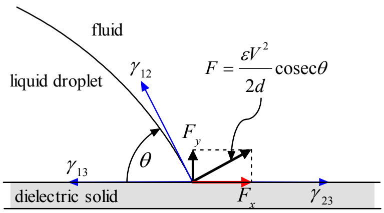

Charge distribution




Maxwell stress

## Surface force and wetting tension

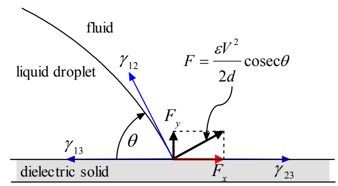
$$F=\int_{S_{12}}\frac{\sigma^2}{2\varepsilon}dl=\frac{\varepsilon V^2}{2d^2}\int_{S_{12}}\frac{1}{(e^u-1)^{2\alpha}}dl=\frac{\varepsilon V^2}{2d}\operatorname{cosec}\theta$$




$$F_x=\frac{\varepsilon V^2}{2d}$$
$$F_y=\frac{\varepsilon V^2}{2d}\cot\theta$$

## Derivation of Lippmann–Young eqn.

- **Conventional electrowetting equation is recovered.**




$$F_{ex}=\frac{\varepsilon V^2}{2d}$$
$$F_{ey}=\frac{\varepsilon V^2}{2d}\cot\theta$$

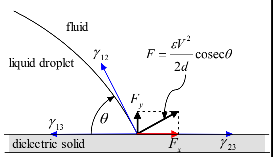
$$\gamma_{12}\cos\theta=\gamma_{23}-\gamma_{13}+\frac{\varepsilon V^2}{2d}$$ ; Lippmann–Young eqn.

(*Langmuir* **2002**, *18*, 10318)

## Origin of electrowetting phenomena

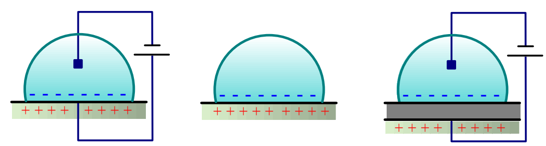
- **Electrowetting originates from wetting tension, rather than from change of surface energy.**
- **Roll of the vertical force should be explained.**

$$F_x=\frac{\varepsilon V^2}{2d},\quad F_y=\frac{\varepsilon V^2}{2d}\cot\theta$$




**Note: Effect of interfacial shape is not considered.**

## Generalization of Theory



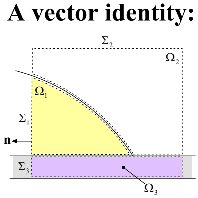
(a) (b) (c)

## Electromagnetic momentum conservation

- **Force on a volume Ω:** $\mathbf{F}^{\Omega}=\int_{\Omega}\mathbf{f}\,dV=\int_{\Sigma}\mathbf{T}\cdot\mathbf{n}\,dS$
- **Maxwell stress with osmotic pressure:**

$$\mathbf{T}=-(\Pi+\tfrac{1}{2}\varepsilon E^2)\mathbf{I}+\varepsilon\mathbf{EE};\qquad \Pi=2n^{\infty}kT[\cosh\beta\varphi-1]$$

- **A vector identity:** $\int_{\Sigma}(\mathbf{n}\cdot\mathbf{E})\mathbf{E}\,dS=\int_{\Omega}[\mathbf{E}(\nabla\cdot\mathbf{E})+\mathbf{E}\cdot\nabla\mathbf{E}]d\Omega$



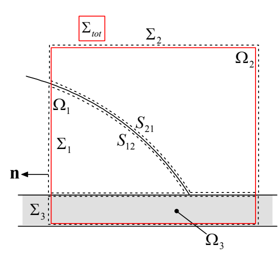

$$\nabla\cdot\mathbf{E}=-\nabla^2\varphi=\frac{\kappa^2}{\beta^2}\sinh\beta\varphi$$

**(Poisson–Boltzmann equation)**

$$\int_{\Sigma}\{[\Pi+\tfrac{1}{2}\varepsilon E^2]\mathbf{I}-\varepsilon\mathbf{EE}\}\cdot\mathbf{n}\,dS=0$$

## Surface force and wetting tension

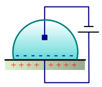
$$\mathbf{F}_{tot}=\mathbf{F}+f_y'\mathbf{e}_y=-f_y'\mathbf{e}_y-\int_{S_{12}+S_{21}}\mathbf{T}\cdot\mathbf{n}\,dS=\int_{\Sigma_{tot}}\mathbf{T}\cdot\mathbf{n}\,dS$$




$$f_x=\frac{\varepsilon_3(V-\varphi_{13})^2}{2d}+\frac{4\varepsilon_1\kappa_1}{\beta^2}\left[\cosh\frac{\beta\varphi_{12}}{2}-1\right]-\frac{\varepsilon_3(V-\varphi_{23})^2}{2d}-\frac{4\varepsilon_2\kappa_2}{\beta^2}\left[\cosh\frac{\beta\varphi_{23}}{2}-1\right]$$

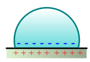
(*Langmuir* **2003**, *19*, 5407)

## Surface force and wetting tension



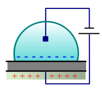
$$W_{el}^{(I)} = 8kT\left(\frac{n_{1b}}{\kappa_1} - \frac{n_{2b}}{\kappa_2}\right)[\cosh\tfrac{\beta V}{2} - 1]$$

Note:
$$\cos\theta = \cos\theta_o + \frac{W_{el}}{\gamma}$$



$$W_{el}^{(II)} = \left\{\frac{8n_{1b}kT}{\kappa_1}[\cosh\tfrac{\beta\varphi_{1\infty}}{2} - 1] - \sigma_1\varphi_{1\infty}\right\} - \left\{\frac{8n_{2b}kT}{\kappa_2}[\cosh\tfrac{\beta\varphi_{2\infty}}{2} - 1] - \sigma_2\varphi_{2\infty}\right\}$$
$$+ (\sigma_1 - \sigma_2)\varphi_o$$

**Geometry dependent term.**
(*Langmuir* **2003**, *19*, 6881)



$$W_{el}^{(III)} = \frac{\varepsilon_3}{2d}[(V - \varphi_{1\infty})^2 - (V - \varphi_{2\infty})^2] + 8\frac{n_{1b}kT}{\kappa_1}[\cosh\tfrac{\beta\varphi_{1\infty}}{2} - 1] - 8\frac{n_{2b}kT}{\kappa_2}[\cosh\tfrac{\beta\varphi_{2\infty}}{2} - 1]$$

## Benefits of the electromechanical theory

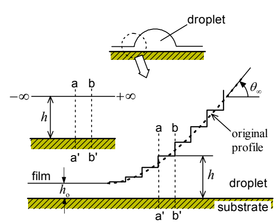
- **Familiar to MEs**
- **Easy and clear; requires understanding only on the Maxwell stress tensor.**
- **Suitable for numerical calculations for complex situations.**
- **Can handle dynamic problems.**

# Usefulness

**Validity of the Derjaguin approximation on the Frumkin–Derjaguin approach**

## Validity of the Derjaguin approximation …

### Frumkine–Derjaguin approach (base on the DLVO theory)



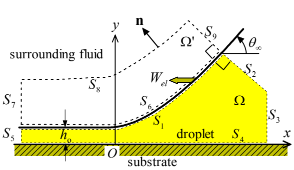

$$\cos\theta_\infty = 1 + \frac{1}{\gamma}\int_{h_o}^{\infty}\pi_t(h)\,dh = 1 + \frac{V_t(h_o)}{\gamma}$$

$$V_{el}^{\varphi}(h_o) = \frac{\varepsilon\kappa}{2}\left[2\varphi_1\varphi_2\operatorname{csch}\kappa h_o - (\varphi_1^2 + \varphi_2^2)(\coth\kappa h_o - 1)\right]$$

$$V_{el}^{\sigma}(h_o) = \frac{\varepsilon\kappa}{2}\left[2\varphi_{1\infty}\varphi_{2\infty}\operatorname{csch}\kappa h_o + (\varphi_{1\infty}^2 + \varphi_{2\infty}^2)(\coth\kappa h_o - 1)\right]$$

## Validity of the Derjaguin approximation …

### Electromechanical approach




$$\cos\theta = \cos\theta_o + \frac{W_{el}}{\gamma}$$

$$W_{el} = -\mathbf{F}\cdot\mathbf{e}_x$$

$$\mathbf{F} = -\int_{S_1}\mathbf{T}\cdot\mathbf{n}\,dS - \int_{S_6}\mathbf{T}\cdot\mathbf{n}\,dS$$

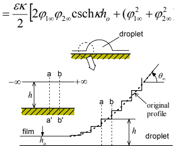
## Validity of the Derjaguin approximation …

### Frumkine–Derjaguin approach (base on the *DLVO* theory)

$$\cos\theta_\infty = 1 + \frac{1}{\gamma}\int_{h_o}^{\infty}\pi_t(h)\,dh = 1 + \frac{V_t(h_o)}{\gamma}$$

$$V_{el}^{\varphi}(h_o) = \frac{\varepsilon\kappa}{2}\left[2\varphi_1\varphi_2\operatorname{csch}\kappa h_o - (\varphi_1^2 + \varphi_2^2)(\coth\kappa h_o - 1)\right]$$

$$V_{el}^{\sigma}(h_o) = \frac{\varepsilon\kappa}{2}\left[2\varphi_{1\infty}\varphi_{2\infty}\operatorname{csch}\kappa h_o + (\varphi_{1\infty}^2 + \varphi_{2\infty}^2)(\coth\kappa h_o - 1)\right]$$



### Electromechanical approach

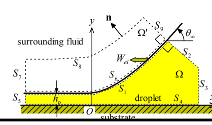
$$\cos\theta = \cos\theta_o + \frac{W_{el}}{\gamma}$$

$$W_{el} = -\mathbf{F}\cdot\mathbf{e}_x$$

$$W_{el}^{\varphi} = \frac{\varepsilon\kappa}{2}\left[2\varphi_1\varphi_2\operatorname{csch}\kappa h_o - (\varphi_1^2 + \varphi_2^2)(\coth\kappa h_o - 1)\right] + \frac{\varepsilon\kappa}{2}\varphi_2^2(\cos\theta_\infty - 1)$$

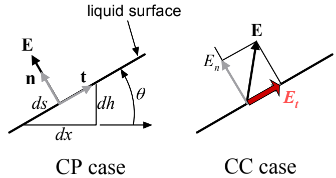
electrocapillary term

$$W_{el}^{\sigma} = \frac{\varepsilon\kappa}{2}(\varphi_{1\infty}^2 - \varphi_{2\infty}^2)(\coth\kappa h_o - 1) + \frac{\varepsilon\kappa}{2}\varphi_{1\infty}^2(\cos\theta_\infty - 1)$$



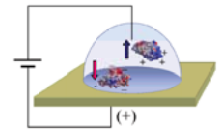
## Validity of the Derjaguin approximation …

- For constant potential (CP) case, valid for *all* the contact angles with minor correction.
- For constant charge (CC) case, significant error due to existence of tangential stress.




## Some of current issues on electrowetting

- Reduction of actuation voltage: now, normally ~30V
- Delay of saturation angle by AC voltage (dielectrophoresis, Prof. Jones in U. Rochester)
- Mechanism of contact angle saturation.
- Contact line instability (droplet ejection)
- Minimization of protein adsorption by additives.




## Concluding remarks

- Electrowetting has many advantages for microfluidic actuation of liquids such as
  - ✓ fast
  - ✓ low energy consumption
  - ✓ digitized operations
- An electromechanical framework on electrowetting is established.